COMPETIFY HUB

FUELING AMBITION, FORGING PATHS

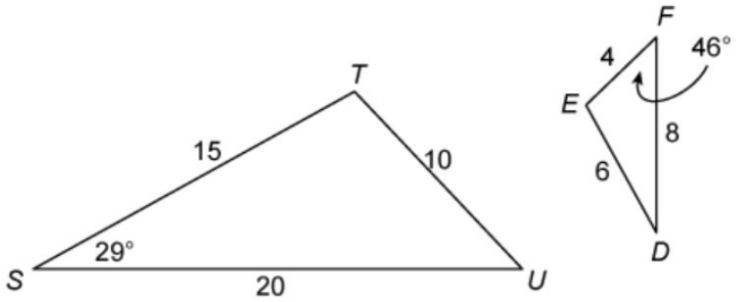
MATH LEVEL 2

GRADES 5-8

PROVIDING FREE RESOURCES FOR ALL

Demo Set 3

Q21: Problem: If triangles STU and DEF are similar, what is the angle measure of angle U?



Q22:Anika is at a birthday party. It is now lunch time, and she must choose what meal she wants. She can choose one drink, two entrees and one dessert. If the order of the food items does not matter, how many different combinations of meals can Anika make?

- Drinks: Water, soda or juice
- Entrees: Pizza, pasta, burgers or fried rice
- Dessert: Cupcakes, brownies, cookies or pudding

S21:Even though similar triangles have different sizes and side lengths, angle measure does not change. Therefore angle U is just 46 degrees because it corresponds with angle F.

S22:Firstly, let's get the easy part out of the way. If Anika can only pick one dessert and one drink, then there are only 12 possibilities, since 3 x 4 = 12. For the entrees, Anika has 4 choices for the 1st entree, and 3 for the 2nd entree. However, the trick is since order doesn't matter, we must divide by 2. If we don't, we count "pizza and pasta" and "pasta and pizza" as two different choices, when they're really the same choice. Therefore 3*4/2 = 6, and 12 x 6 = 72. So the total number of possible meal choices is 72.

Q23:Fred is playing a card game with a normal stack of cards (52 cards + 2 Jokers = 54 cards). The goal of this game is to draw the 2 Jokers in two tries (one card per try). However, this card game is also rigged, as every time you draw a Joker, it has a 1 in 3 chance of disappearing from your hand and back into the deck. Calculate the chances of you drawing both Jokers in two draws without them disappearing from your hand.

Q24: Problem: Javier is reading a book, when he stumbles upon the sentence FERRARIS ARE COOL. Given that the spaces between each word don't count for anything and that the letters of each word do not mix with each other, how many permutations are possible with the sentence FERRARIS ARE COOL?

S23:Find the probability of drawing the first Joker, which is simply 254 since there are 2 Jokers and 54 cards. If you successfully draw it, then it has a 23 of staying in your hand. Drawing the second Joker has a 153 chance, since there is one Joker out of 53 cards remaining. This Joker also has a 23 chance of staying in your hand. After that, just calculate your chances by multiplying all the fractions.

(2/54)*(⅔)*(1/53)*(⅔)=4/12879

Thus, the chances of you drawing both Jokers without them disappearing from your hand is 4/12879.

S24:Not counting spaces, nor mixing the words, will give us the idea we need to calculate the possible permutations for each word individually. The formula for permutations is n!/(n1! · n2! · … · nk!) Where n! is all the possible permutations, and n1!, n2! is how many times each element in the word repeats. Therefore to find each word, we just plug the values in:

FERRARIS: 8!/3! = 6,720. 8! represents all possible permutations, while 3! accounts for the 3 Rs in the word. All other letters appear once, and since 1! = 1, we can leave them out since they wouldn't change the equation anyway.

ARE: Since ARE doesn't have any repeating letters, its permutations are just equal to 3!, which is equal to 6.

COOL: 4/!2! = 12. Once again, 4! accounts for all possible permutations, and 2! accounts for the 2 Os in the word. Once again, 1! = 1, so we leave them out.

Therefore, to find the total number of possible permutations, we multiply 6720 x 6 x 12, which is equal to 483, 840. Thus, there are 483, 840 permutations in the sentence FERRARIS ARE COOL.

Q25:In the range of numbers between 100 and 999 (inclusive) a number is considered *groovy* if it meets the following 3 requirements:

- It is a palindrome
- It is divisible by 11
- The sum of its digits is also greater than 11

How many numbers in the total range can be considered *groovy*?

Q26:A professor is reading a book. He flips to a random part of the book and finds out that the product of the page numbers facing him is equal to 4032. Find the sum of the two pages he flipped to. (Note that all page numbers are numbered consecutively.)

S25:To first shrink down our numbers, we must find all 3 digit palindrome numbers. 90 3 digit palindromic numbers exist. (A palindromic number is a number that is the same if its digits are reversed, such as 101, 111, 121, etc) We know this because there are exactly 10 palindromes for every 100 numbers, so 10 x 9 = 90. We can use the third rule, the sum of its digits are greater than 11, to eliminate some of the numbers. After doing some casework, we are left with 60 numbers. That's still a lot, but we can use the divisibility rule of 11 to check the remaining 60 numbers. The divisibility rule of 11 states that if the difference of the sum between the digits at the odd positions and even positions are equal to 0 or 11, then that number is divisible by 11. After some more casework, we are left with 6 *groovy* numbers: 363, 484, 616, 737, 858 and 979.

S26:Firstly, we can narrow down the scope of the numbers using much friendlier numbers. We know that 60 x 60 = 3600 and 70 x 70 = 4900, which immediately gives us the idea that the two numbers are between 60 and 70. You can either brute force from here since 4032 seems to be perfectly smack in the middle (meaning you can try 64 or 65), or try to find the square root of 4032, which you'll find to be slightly less than 64. Either way you do it, your final answer should be 63 + 64 = 127 (since 63 x 64 = 4032), which gives you 127 as the correct answer.

Q27:Rudy the Rabbit has 123 carrots and began peeling carrots at a rate of 6 carrots per minute. 4 minutes later, Carrol the Rabbit joined her and began peeling at a rate of 5 carrots per minute. When they finished, how many carrots had Rudy the Rabbit peeled?

Q28:A pile of earth removed from an excavation is a cone measuring 9 ft high and 24 ft across its base. How many trips will it take to haul away the earth using a dump truck with a capacity of 9 cubic yards? Round your final answer to the nearest number.

S27:First we calculate how many carrots Rudy peeled by herself. 6 x 4 = 24 carrots. Which means when Carrol joined, they had 99 carrots left (123 - 24). Together, the two of them are peeling carrots at a rate of 11 carrots per minute. So they finish peeling in 9 minutes (99/11), which means that 6 x 9 = 54 carrots Rudy peeled while with Carrol. Plus the 24 carrots she peeled while by herself, Rudy the Rabbit peeled 54 + 24 carrots, which equals to 78 carrots.

S28:Before we start this problem, it is important to notice that the dimensions of the cone are measured in feet, but the problem is asking us to find the volume in cubic yards, since the dump truck is measured in that unit. Therefore, we must change our values before we plug them in the formula for a cone's volume. For the height, there are 3 yards in 9 feet. To find the radius, simply divide 24 by 2, giving us 12 feet, which equals to 4 yards. (1yd = 3ft) Plugging in our numbers measured by yards gives us 13π(4)^2(3) which equals 50.24 cubic yards. Divide this number by 9, which gives us 5.582 (repeating.) In this problem, we round up, so it will require the dump truck 6 trips to haul away all the dirt.

Q29:A plane intersects a sphere with a volume of 113.1 cubic meters. What is the area of the cross section? Use 3.14 for π. Round to the nearest tenth. (Any other numbers you calculate may also be rounded to the nearest tenth.)

Q30: A store sells solid metal support poles in the form of right cylinders that are made out of metal with a density of 5.7 grams per cubic centimeter. This metal can be purchased for $0.27 per kilogram. Calculate the cost of one of these utility poles with a diameter of 41.4 cm and a height of 680 cm. Use 3.14 for π. Round your answer to the nearest cent. (Any other numbers you calculate may also be rounded to the nearest hundredth.)

S29: Let's start by understanding what we're supposed to find. A cross section in the center of a sphere is usually a circle. The formula for a circle is πr^2 ,however we must solve for *r* first. To do that, we use the formula of a sphere to set $113.1 = 43\pi r3$. Then we solve for *r* by multiplying ¾ (to cancel out the 4/3), divide by π and find the cube root of that number to be roughly around 3. Thus r = 3, and plugging it back into our equation for the area of a circle gives us 28.3 square meters (rounded to the nearest tenth).

S30: Our first step should be to find the volume of the cylinder it is asking for, before working out either the mass (density) or cost of the pole. The formula for the volume of a cylinder is $\pi r2h$- so to find the volume, we divide the diameter by half (41.4/2 = 20.7). Then we plug the values in to find the volume of the cylinder is 914911.85 cubic centimeters. Next, we calculate the mass of the cylinder. To do this, we just multiply the volume by 5.7, since the density is 5.7 grams per cubic centimeter. We get 5214997.55 cubic centimeters. However, we must realize before we do the next step, that the cost is calculated in KILOGRAMS, not grams. There are 1000 grams in a kilogram, which means we must divide by 1000 before finding the cost. This gives us 5215 cubic kilograms. Finally, we multiply 0.27 (the cost) by our mass in kilograms, to find that the answer is $1408.05.